

Entanglement evolution for excitons of two separate quantum dots in a cavity driven by magnetic field

Jun Jing*, Z. G. Lü, H. R. Ma

Institute of Theoretical Physics, Shanghai Jiao Tong University

800 DongChuan Road, MinHang, Shanghai 200240, China

(Dated: February 1, 2008)

Abstract

The time evolution of entanglement for excitons in two quantum dots embedded in a single mode cavity is studied in a “spin-boson” regime. It is found that although with the dissipation from the boson mode, the excitons in the two quantum dots can be entangled by only modulating their energy bias ϵ under the influence of external driving magnetic field. Initially, the two excitons are prepared in a pure separate state. When the time-dependent magnetic field is switched on, a highly entangled state is produced and maintained even in a very long time interval. The mechanism may be used to control the quantum devices in practical applications.

PACS numbers: 03.67.-Lx, 85.30.Vw, 42.50.-p

* Email: jingjun@sjtu.edu.cn

I. INTRODUCTION

Since the age of *EPR* paradox [1], quantum entanglement, as the magic non-local correlation of quantum system revealed by the violation of Bell's inequality [2], had attracted many attentions. During the past two decades, entangled states provide great potential in the quantum communication and information process [3, 4]. It is also regarded as the most intriguing and inherent feature of quantum composite system and the resource of quantum computation [5]. Thus the preparation of entangled pair in a practical way and the maintenance of its entanglement degree in a comparatively long time are important issues for the realization of qubits and quantum gate.

There is significant interest in quantum information processing using semiconductor quantum dot. Recently, there are many works devoted to the quantum information properties, such as the entanglement creation, of semiconductor quantum dot, since it is a qualified qubit candidate [6]. In comparison with other physical systems [7], solid-state devices, in particular, ultra-small quantum dots [8] with spin degrees of freedom embedded in nanostructured materials are more easily scaled up to large registers and they can be manipulated by energy bias and tunneling potentials [9]. Burkard et al. [8] consider a quantum-gate mechanism based on electron spins in coupled quantum dots, which provides a general source of spin entanglement. Liu et al. [10, 11] investigated the generation of the maximally entanglement for coherent excitonic states in two coupled large quantum dots ("large" means $R \gg a_B$, where R is the radius of each dot and a_B is Bohr radius of excitons in quantum dots) mediated by a cavity field, which is initially prepared in an odd coherent state. Zhang et al. [12] showed that the Bell states and GHZ states can be robustly generated by manipulating the system parameters. Chen et al. [13] observed that the photon trapping phenomenon in double quantum dots (their distance is small compared to the emitted photon λ) generates a entangled state. Nazir et al. [14] found that two initially nonresonant quantum dots may be brought into resonance by the application of a single detuned laser. This allowed for the generation of highly entangled excitonic states on the 10^{-12} second time scale. The possibility of creating spin quantum entanglement in a system of two electrons confined respectively in two vertically coupled quantum dots in the presence of spin-orbit interaction had been explored by Zhao et al. [15]. However, most of

their successful works were devoted to producing entanglement from two coupled quantum dots, but there are few schemes to create a highly entangled state for excitons in separate quantum dots from a pure separate state by only external periodical field.

Cavity QED experiments, where few atoms are coupled to single cavity modes, have culminated in the demonstration of creation of entanglement between three distinguishable quantum systems [16]. Indeed an integrated cavity QED consisted of two quantum dots is more practical and realizable [17]. This paper is applied for the generation of an entangled excitonic state in the system of two identical quantum dots placed in a single-mode cavity. The number of electrons excited from the valence-band to the conduction-band in each dot is assumed to be small and the exciton-exciton interaction in the same dot can be neglected [18]. Then the subsystem in which we hope to create entanglement is consisted of two exciton picked from the two quantum dots respectively. Each of them are simplified by considering only the ground state and the first excited state of it. Then the whole system is approximated like a spin-boson model. The distance of the two dots is large enough to ignore the direct coupling between two excitons and they are connected only through the cavity mode. The measurement of the subsystem was chosen to be the concurrence found by Wootters' group [19, 20] in the year of 1997 and von Neumann entropy. It will be demonstrated that in our model, the entanglement of the two excitons can be enhanced to a high degree by the external field. And we also try to clarify the physics behind it. The rest of this paper is organized as follows. In section II, we introduce the Hamiltonian for our "spin-boson" model and describe the computation procedure for the time-evolution of the concurrence and quantum entropy for the subsystem; Detailed results and discussions are in section III; The conclusion of our study is given in section IV.

II. THE HAMILTONIAN AND THE THEORY

Essentially, the model we studied is two separate two-level atoms (spins) connected by a single-mode boson gas under an periodical magnetic field along \vec{z} direction. We assume that the the two "atom" is identical and their interactions with the boson are identical. The

Hamiltonian of this model can be written as:

$$H = H_0 + H_t, \quad (1)$$

$$H_0 = \sum_{i=1}^2 \left(-\frac{\Delta_i}{2} \sigma_i^x + \frac{\epsilon_i}{2} \sigma_i^z \right) + \omega \left(a^\dagger a + \frac{1}{2} \right) + g \sum_{i=1}^2 (a + a^\dagger) \sigma_i^x, \quad (2)$$

$$H_t = \sum_{i=1}^2 F(t) \sigma_i^z, \quad (3)$$

where ϵ is the energy bias and Δ is the tunneling potential. The time-dependent periodical (the period is P) external field $F(t)$ could be a rectangular wave, a triangular wave or a cosine wave $F(t) = A \cos(\frac{2\pi}{P}t)$, wherein A is the amplitude, P is the period length. ω is the frequency of the single-mode and $\omega \ll \epsilon$. g is the spin-boson coupling strength and $g \ll 1.0$. σ_x and σ_z are the well-known Pauli matrix:

$$\sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (4)$$

The whole state of the whole system $\rho(t)$ can be formally calculated by such a process:

$$\rho(t) = \exp(-iHt) \rho(0) \exp(iHt), \quad (5)$$

$$\rho(0) = \rho_{12}(0) \otimes \rho_b(0), \quad (6)$$

$$\rho_{12}(0) = |\psi(0)\rangle \langle \psi(0)|, \quad (7)$$

$$\rho_b(0) = |\phi(0)\rangle \langle \phi(0)|, \quad (8)$$

where the initial state of the two spins $\psi(0)$ is a separate state, for instance, $\psi(0) = |0\rangle_1 |1\rangle_2$, both $|0\rangle$ and $|1\rangle$ are the eigenstates in the space of $\{\sigma^2, \sigma^z\}$; $\phi(0)$ is the initial state of the boson mode, which is set as a vacuum state $|0\rangle$. The evolution operator $U(t) = \exp(-iHt)$ can be evaluated by the efficient algorithm of polynomial schemes [22, 23, 24]. The method used here is the Laguerre polynomial expansion method we proposed in Ref. [24], which is pretty well suited to this problem and can give accurate result in a comparatively smaller computation load. More precisely, the evolution operator $\exp(-iHt)$ is expanded in terms of the Laguerre polynomial of the Hamiltonian as:

$$U(t) = e^{-iHt} = \left(\frac{1}{1+it} \right)^{\alpha+1} \sum_{k=0}^{\infty} \left(\frac{it}{1+it} \right)^k L_k^\alpha(H),$$

where α distinguishes different types of Laguerre polynomials [25], k is the order of the Laguerre polynomial. In real calculations the expansion has to be cut at some value of

k_{\max} , which was taken to be 20 in this study. With the largest order of the expansion fixed, the time step t is restricted to some value in order to get accurate results of the evolution operator. For longer times the evolution can be achieved by more steps. The action of the Laguerre polynomial of Hamiltonian to the states is calculated by recurrence relations of the Laguerre polynomial. The efficiency of it is about 8 times faster than conventional methods such as Runge-Kutta algorithm.

After obtaining the density matrix of the whole system, the reduced density matrix of the two two-level excitons can be found by a partial trace operation to $\rho(t)$, which traces out the degrees of freedom of the single-mode boson:

$$\rho_{12}(t) = \text{Tr}_b(\rho(t)). \quad (9)$$

For the model of this paper, $\rho_{12}(t)$ can be expressed as a 4×4 matrix in the Hilbert space of the subsystem spanned by the orthonormal vectors $|00\rangle$, $|01\rangle$, $|10\rangle$ and $|11\rangle$.

The main tool of discussing pairwise entanglement is the concept of concurrence. The definition of concurrence between spins 1 and 2 can be found in [19, 20]:

$$C = \max\{\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0\}, \quad (10)$$

It is a monotone of von-Neumann entropy. λ_i , $i = 1, 2, 3, 4$ are the square roots of the eigenvalues of the product matrix $\rho_{12}\tilde{\rho}_{12}$ in decreasing order. Where $\tilde{\rho}_{12}$ is constructed as $(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$. Equation (10) applies for both of the mixed state and pure state. For a maximally entangled state, $C = 1$, and for a separate one, $C = 0$.

III. RESULTS AND DISCUSSIONS

In this section, we showed the time evolutions of concurrence for ρ_{12} under different kinds of periodical magnetic fields in comparison with that with no field. The energy bias and tunneling potential in Hamiltonian (1): $\epsilon = \Delta = 0.4$. The amplitude A and the period P of rectangular wave, triangular wave and cosine wave are $A = 0.48, P = 4.0$. The other parameters are kept as: $\omega = 0.02, g = 0.02$.

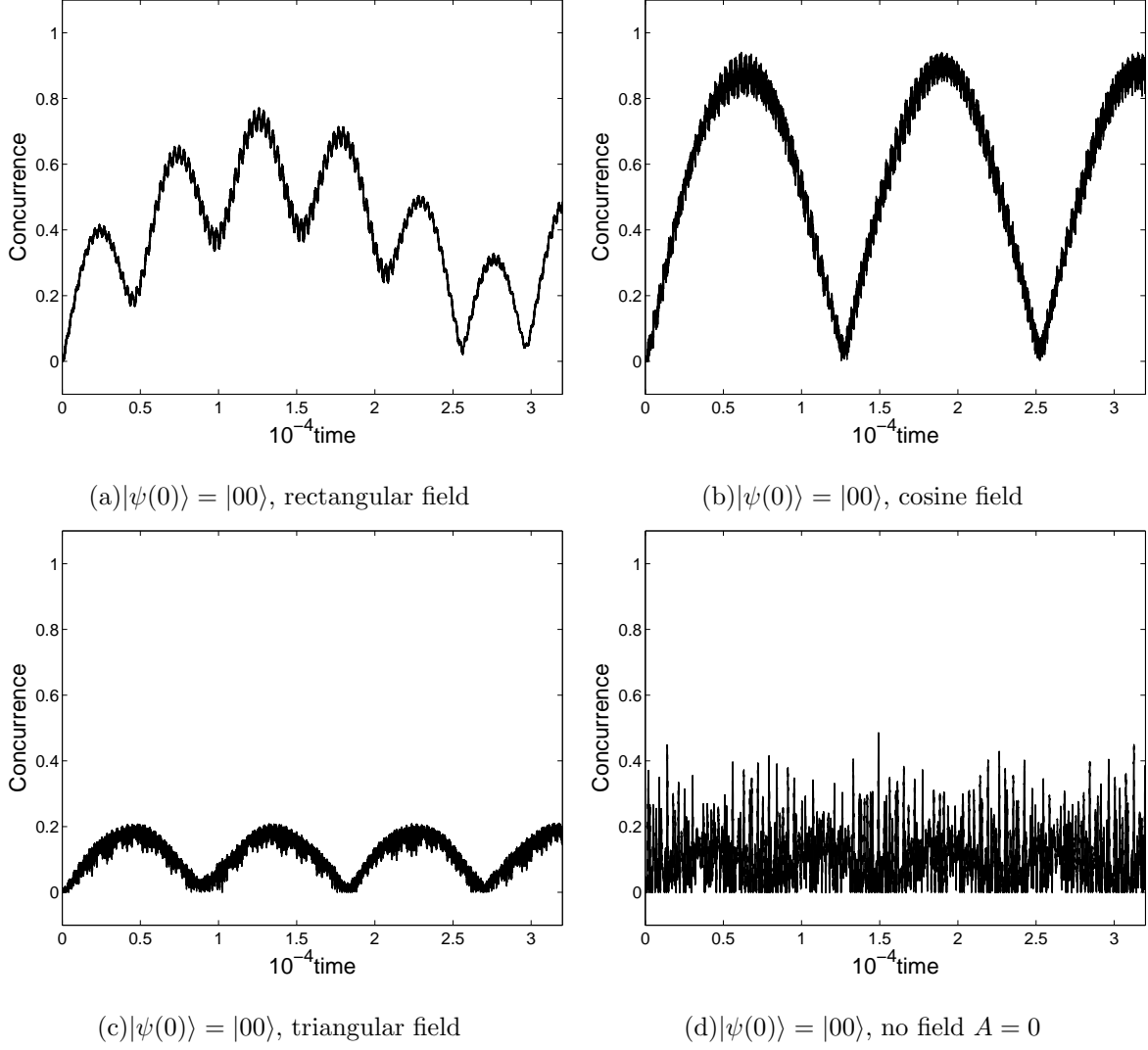


FIG. 1: The time-evolution of entanglement for $|\psi(0)\rangle = |00\rangle$ under different field vs. the pure dissipation process without any modulating action

The calculated entanglement evolution are plotted in the figures of 1, 2, 3 according to different kinds of modulating field with different initial condition for the two excitons in quantum dots $|\psi(0)\rangle$. Fig. 1 shows the concurrence for the initial pure state $|\psi(0)\rangle = |00\rangle$ under rectangular wave 2(a), cosine wave 2(b), triangular wave 2(c) and without external field 2(d). Obviously, the entanglement fluctuations in sub-figure 2(d) are greatly decreased by the periodical external field and the entanglement evolution is improved to different extent. The effect of cosine wave 2(b) is better than the others. And Fig. 2 and Fig. 3 display the concurrence for initial condition of $|\psi(0)\rangle = |01\rangle$ and $|\psi(0)\rangle = |11\rangle$, respectively. It is easy to see that $|01\rangle$ and $|00\rangle$ or $|11\rangle$ can be distinguished by the concurrence evolution

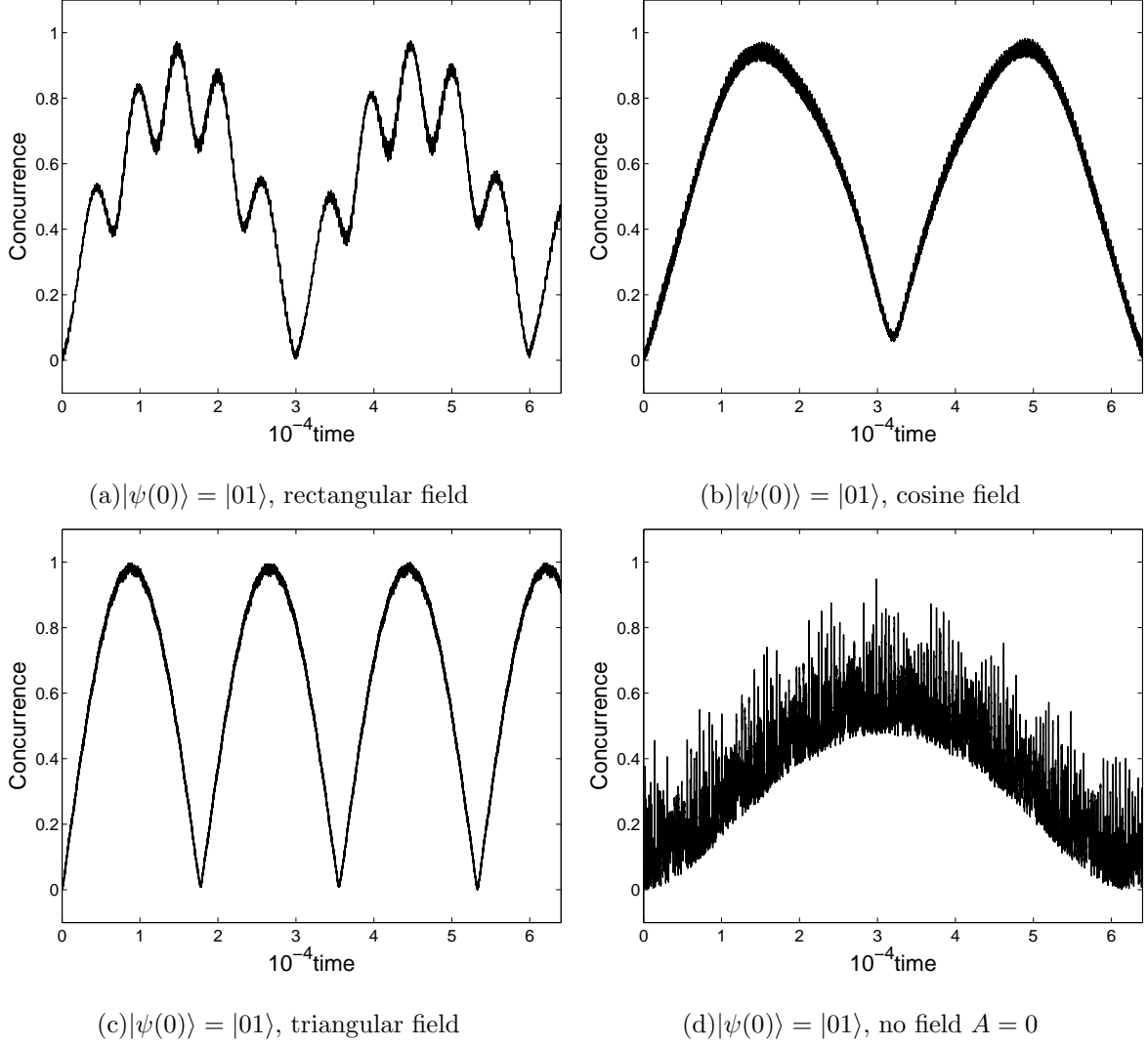


FIG. 2: The time-evolution of entanglement for $|\psi(0)\rangle = |01\rangle$ under different field vs. the pure dissipation process without any modulating action

driven by the same magnetic field.

Through the driving of the magnetic field, a much steady and high entanglement between the two quantum dots can be prepared. It results from the adjustment of the energy bias due to the field $F(t)$. In some interval of one period of $F(t)$, the energy bias is reduced, so that the probability of the exciton transition is greatly increased due to the Poisson probability distribution of the multiple photon process [26, 27]. Then the probability that the two excitons make transition simultaneously is also much greater than that without such a periodical field. When this kind of process happens, the two excitons gain some

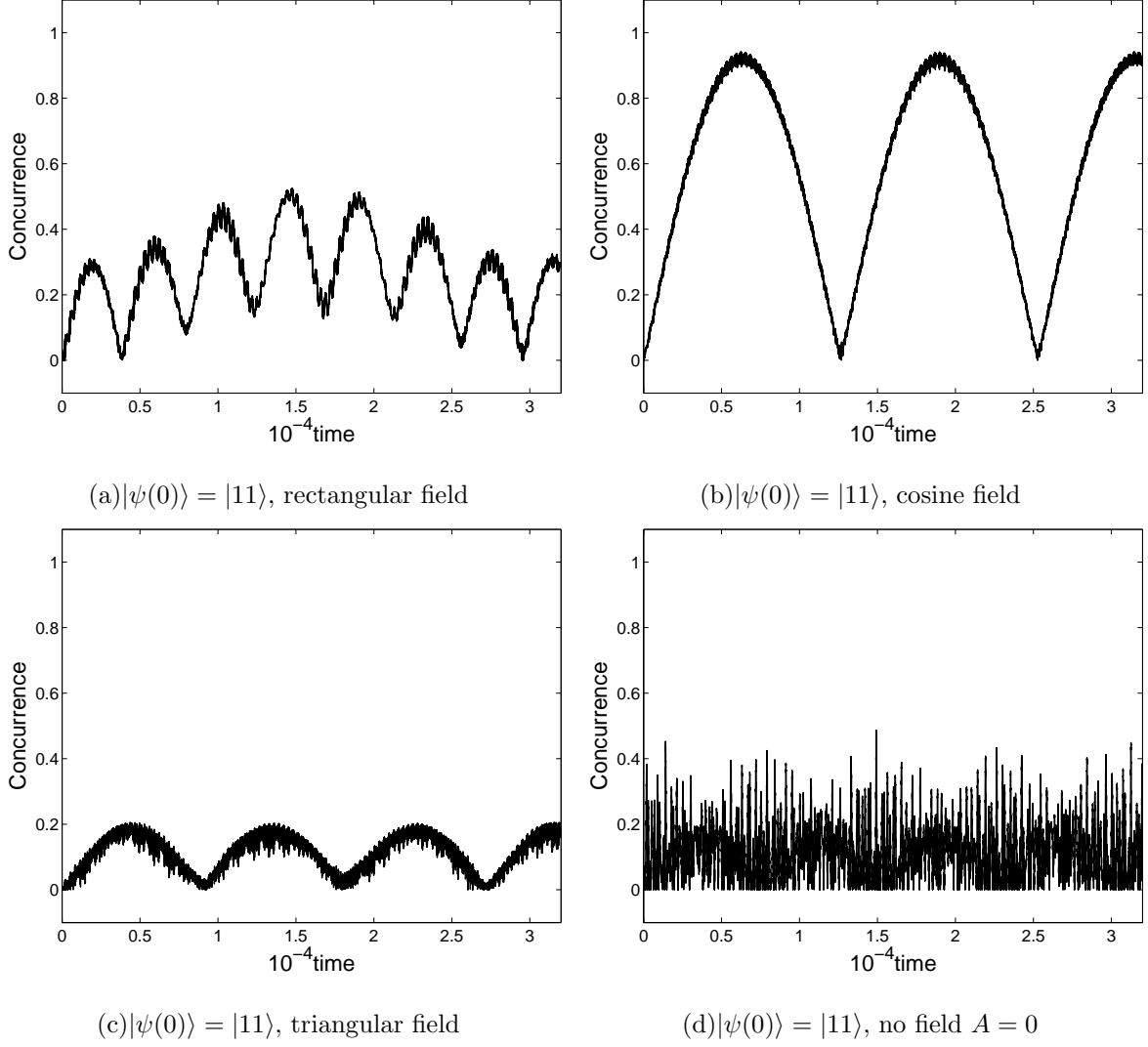


FIG. 3: The time-evolution of entanglement for $|\psi(0)\rangle = |11\rangle$ under different field vs. the pure dissipation process without any modulating action

spatial correlation. The correlation helps to increase entanglement between them. And in other interval of the same period, the energy bias is larger than that without such a field, it could help to maintain the entangled state just gained by the above correlation process. At the same time, the spontaneous radiation happens randomly which reduces the concurrence.

Three important features need to be noticed: (i) The effect of the single-mode cavity, $g \sum_{i=1}^2 (a + a^\dagger) \sigma_i^x$ in equation 1, is twofold. On the one hand, indirect interaction between the two excitons is induced by cavity mode. If both excitons could transfer synchronously, they are correlated at the same time, which makes them further entangled in space.

However that probability is very small due to the condition $\omega \ll \epsilon$. On the other hand, it is quantum noise [28] that effects on the entangled pair. Therefore, there are great fluctuations in the time region as in the last sub-figure of figures 1, 2, 3. (ii) There is evident difference between the evolution results from initial state $|01\rangle$ and $|00\rangle$ or $|11\rangle$. For the initial state of $|01\rangle$, we find all the three kinds of periodical fields can create and maintain a highly entangled exciton-pair: in figure 2(a), the peak value of the first wave envelope is 0.971332 and around the same peak, the fidelity of $C \geq 0.5$ persists a sufficient long time 14792; in figure 2(b), the first peak value is 0.971071 and around the same peak, the time interval of fidelity $C \geq 0.5$ is as long as 20389; in figure 2(c), the first peak value is 0.997605 and around the same peak, the time interval of $C \geq 0.5$ is 10899. This advantage of the case $\psi(0) = |01\rangle$ over the other two cases stems from the effect of the single-mode cavity during the exciton transition. When the system is prepared in $|01\rangle$ or $|10\rangle$, one exciton could drop from the excited level and simultaneously the other exciton could absorb the photons just emitted by the former. This is a virtual process by which the two excitons are correlated. But when the system is in $|00\rangle$ or $|11\rangle$, this kind of synchronization resorts to the photons provided by the single-mode. That is a true process whose probability is much lower. (iii) The rectangle wave or triangle wave essentially consists of infinite cosine waves of multiply frequencies. Thus the combined effect of entanglement enhancement from them is not as good as that from a fundamental-frequency cosine wave due to their asynchronous evolution (to compare sub-figure (b) with (a) or (c) in figure 1, 2, 3).

Since entanglement entropy can measure the amount of quantum information inside an entangled pair of qubits, we give the time evolution results of entropy for the initial pure state $|\psi(0)\rangle = |01\rangle$ under rectangular wave 4(a), cosine wave 4(b), triangular wave 4(c) and without external field 4(d). It is evident that the evolution behavior of the von Neumann entropy is in good agreement with that of the concurrence. It is further proved that highly-entangled pair of qubits can be created by controlled external field.

IV. CONCLUSION

In this paper, a entangled exciton pair is prepared in two separate quantum dots embedded in a single-mode cavity by a periodical external magnetic field. The two excitons are

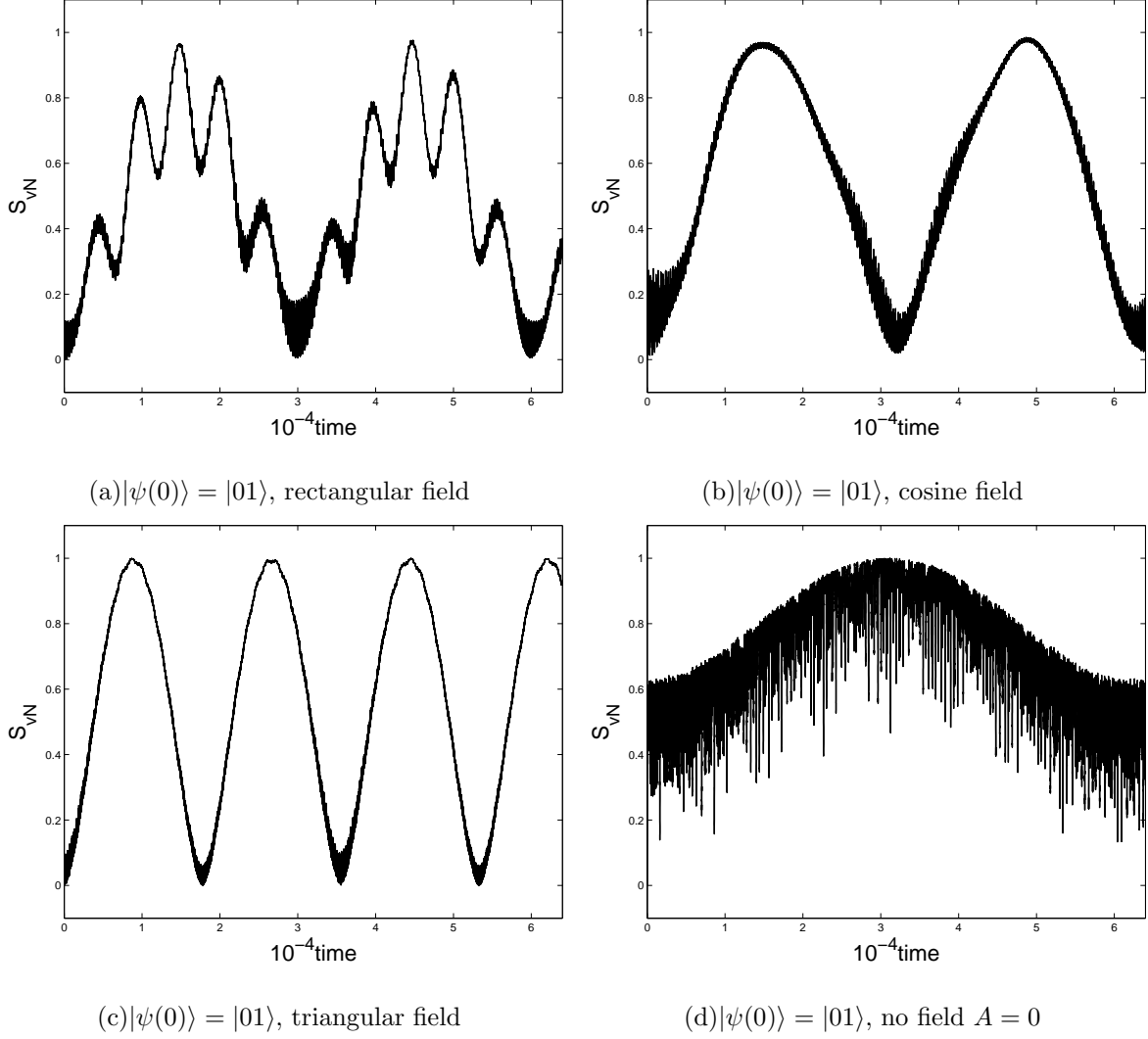


FIG. 4: The time-evolution of von Neumann entropy for $|\psi(0)\rangle = |01\rangle$ under different field vs. the pure dissipation process without any modulating action

simplified as two two-level atom or spins, which are initially prepared as a product state, which means $C(\psi(0)) = 0$. The calculated results suggest that it is possible to create steady highly-entangled pair by a driving external field. And we find that an almost maximally entangled state can be prepared from $\psi(0) = |01\rangle$ or $\psi(0) = |10\rangle$. The results indicated that the cosine wave has advantage over the rectangular and triangular field in entanglement enhancement and persistence. Our approach can be used to study the evolution of entanglement and quantum entropy for two quantum dots. The revealed physical mechanism can

be applied to control the quantum devices in the future.

- [1] A. Einstein, B. Podolsky, N. Rosen, Phys. Rev. 47, 777 (1935)
- [2] J.S. Bell, Physics. 1, 195 (1964)
- [3] C.H. Bennett, H.J. Bernstein, S. Popescu, B. Schumacher, Phys. Rev. A 53, 2046 (1996)
- [4] M. Nielsen, I. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, England, 2000)
- [5] L.K. Grover, Phys. Rev. Lett. 79, 325 (1997)
- [6] S.S Li, J.B Xia, et. al, J. Appl. Phys. 90, 6151 (2001)
- [7] S. Braunstein, and H.K. Lo, Eds., Fortschr. Phys. 48, 765 (2000)
- [8] G. Burkard, D. Loss and D.P. DiVincenzo, Phys. Rev. B 59, 2070 (1999)
- [9] D. Loss, and D.P. DiVincenzo, Phys. Rev. A 57, 120 (1998)
- [10] Y.X. Liu, S.K. Özdemir, M. Koashi and N. Imoto, Phys. Rev. A 65, 042326 (2002)
- [11] A. Miranowicz, S.K. Özdemir, Y.X. Liu, M. Koashi, N. Imoto and Y. Hirayama, Phys. Rev. A 65, 062321 (2002)
- [12] P. Zhang, C.K. Chan, Q.K. Xue and X.G. Zhao, Phys. Rev. A 67, 012312 (2003)
- [13] Y.N. Chen, D.S. Chu and T. Brandes Phys. Rev. Lett. 90, 166802 (2003)
- [14] A. Nazir, B.W. Lovett and G.A.D. Briggs, Phys. Rev. A 70, 052301 (2004)
- [15] N. Zhao, L. Zhong, J.L. Zhu and C.P. Sun, Phys. Rev. B 74, 075307 (2006)
- [16] J.M. Raimond, M. Brune, S. Haroche, Rev. Mod. Phys. 75, 565 (2001)
- [17] A. Badolato, et al, Science 308, 1158 (2005)
- [18] X.G. Wang et al., Phys. Rev. A 67, 022302 (2003)
- [19] S. Hill, W.K. Wootters, Phys. Rev. Lett. 78, 5022 (1997)
- [20] W.K. Wootters, Phys. Rev. Lett. 80, 2245 (1998)
- [21] R.P. Feynmann and F.L. Vernon, Ann. Phys, N.Y. 24, 118 (1963)
- [22] V.V. Dobrovitski, H.A. De Raedt, Phys. Rev. E 67, 056702 (2003)
- [23] X.G. Hu, Phys. Rev. E 59, 2471 (1999)
- [24] J. Jing, H.R. Ma, Phys. Rev. E, accepted
- [25] G. Arfken, *Mathematical Methods of Physicists*, 3rd ed Academic, New York, (1985)
- [26] L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics*, Cambridge University Press,

Cambridge (1995)

- [27] M.O. Scully and M.S. Zubairy, *Quantum Optics* Cambridge University Press, Cambridge, (1997)
- [28] C.W. Gardiner and P. Zoller, *Quantum Noise: A Handbook of Markovian and Non-Markovian Quantum Stochastic Methods with Applications to Quantum Optics*, Springer-Verlag, Berlin (1999)